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GEOMETRIC PROPERTIES OF A PRESTRESSED SEGMENTED SPHERICAL SHELL

by R. L. Brackett, F. R. Shanley, and L. P. Felton

Prepared by
UNIVERSITY OF CALIFORNIA
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ABSTRACT

Ceramics are potentially excellent structural materials because of their high moduli of elasticity and great compressive strengths. However, their brittleness, coupled with fabrication difficulties, has been a major drawback to their increased use. These problems might be overcome if structures were to be constructed of small segments with the assemblage being prestressed.

This report discusses the design and construction of a plastic model of a prestressed dome constructed of 180 triangular plate segments. Prestressing is accomplished by cables running between segments, parallel to segment edges. Prestressing forces are adjusted by special fittings located around the perimeter of the dome.

I. INTRODUCTION

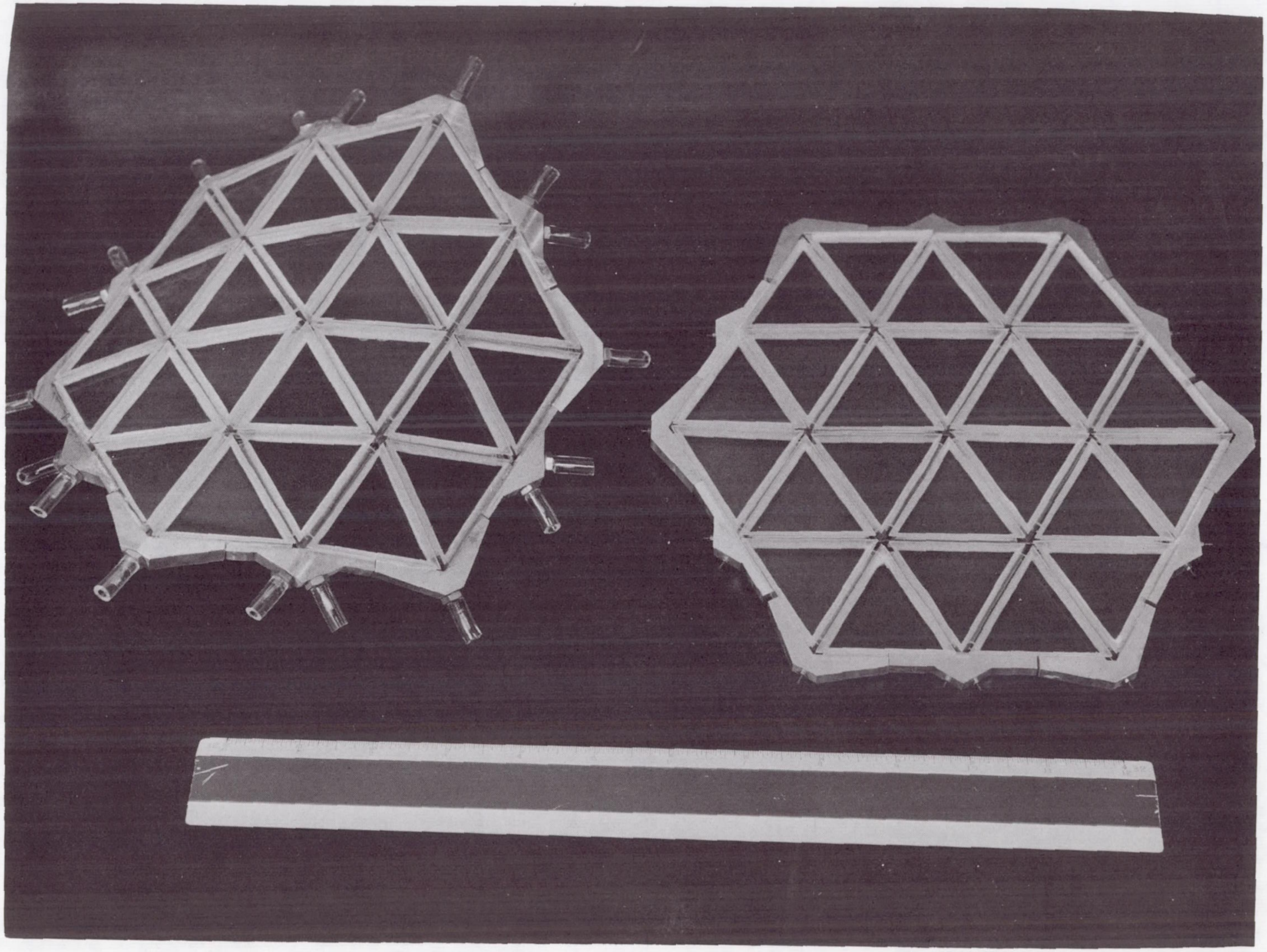
The need for high strength, high temperature materials for aerospace structures has been recognized for many years. The use of ceramics in structural applications has recently received increasing attention.

Ceramics are potentially excellent structural materials because of their high moduli of elasticity, low densities, and great compressive strengths at normal and elevated temperatures. However, their brittleness has made them unsuitable for use as structural members in tension fields. Furthermore, the problems associated with fabricating large components of ceramics have been major drawbacks to their increased use.

It has been proposed by Shanley[1] that many of these difficulties might be overcome if large structures were to be constructed of small segments, the assemblage being prestressed. The prestressing serves the twofold purpose of holding the assemblage together, and of minimizing or eliminating tensile stresses under load by introducing any desired degree of precompression. The use of small segments eliminates warping and cracking of large monolithic structures during fabrication, and prevents possible catastrophic propagation of cracks throughout the structure during service.

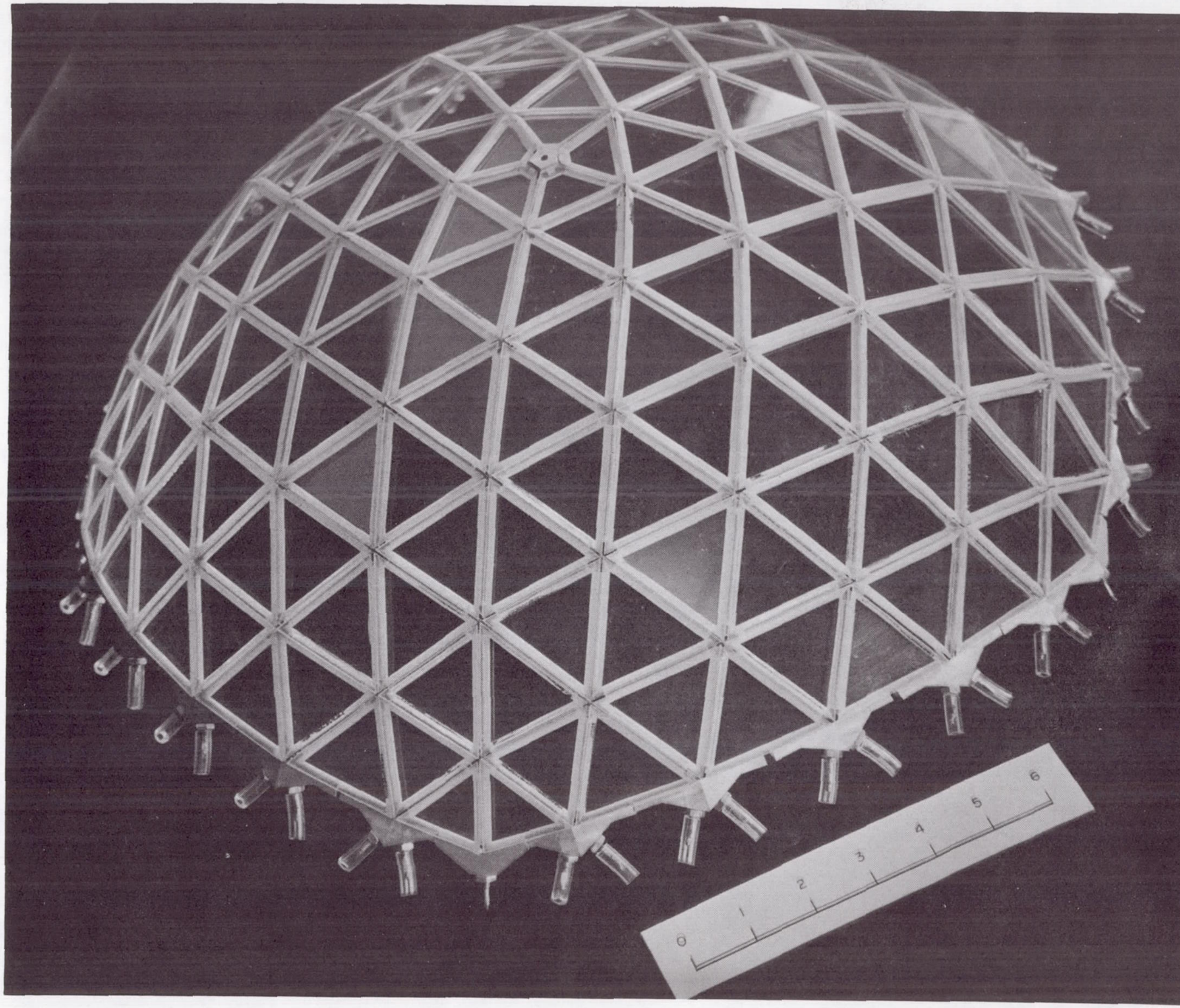
Research currently in progress is directed toward the investigation of the behavior of such relatively large prestressed, segmented ceramic structures. As a preliminary step toward this goal, the geometric properties of several types of structures were studied, and plastic (plexiglass) models were constructed. Figures 1 and 2 show three such models. The flat plate and cylindrical section shown in Figure 1 are developable surfaces of relatively simple geometry. This report deals in detail with the geometric properties and construction details of the spherical dome shown in Figure 2. This spherical shell is typical of complex structural shapes which will ultimately be fabricated of ceramics, and which will have applications such as reentry vehicle heat shields and nose-cones, and as possible deep-submergent underwater vehicles.

Before discussing the actual design requirements and solution, it is appropriate to define some of the terms which will be found in the following sections:



DEMONSTRATION MODELS FOR PRESTRESSED CERAMIC SHELL STRUCTURE SHOWING PORTION OF A FLAT PLATE AND OF A CYLINDRICAL STRUCTURE

FIGURE 1

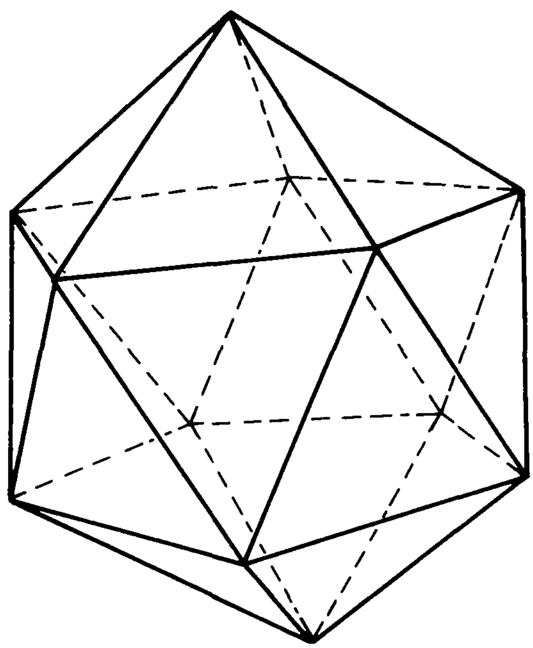


PLASTIC MODEL OF PRESTRESSED DOME

FIGURE 2

GEODESIC — Of or pertaining to great circles of a sphere, or of arcs of such circles; as a geodesic line, hence a line which is a great circle or arc thereof; and as a geodesic pattern, hence a pattern created by the intersection of great circle lines or arcs, or their cords.

ICOSAHEDRON — A polyhedron of twenty faces (see Figure 3).



ICOSAHEDRON

FIGURE 3

SPHERICAL ICOSAHEDRON — An icosahedron "exploded" onto the surface of a sphere; bears the same relation to an icosahedron as a spherical triangle bears to a plane triangle.

ISOSACAP — Five spherical triangles of a spherical icosahedron having a common vertex (see Figure 2).

ISOSCELES — Having two equal sides.

EQUILATERAL — Having all sides of equal length.

SCALENE — Having no sides of equal length.

GREAT CIRCLE — Any circle generated on the surface of a sphere by the intersection of the sphere and a plane which passes through the center of the sphere.

SPHERICAL TRIANGLE — Any triangle which lies on the surface of a sphere.

SPHERICAL ANGLE — The angle generated by the tangents to two arcs drawn on the surface of a sphere.

II. DESIGN OF PRESTRESS PATTERN

As is observable from Figures 1 and 2, the basic segment form, selected for use in all these structures, is the triangle. Triangular segments were selected primarily because it is anticipated that their use in structures will allow prestressing patterns most nearly leading to "quasi-isotropic" behavior under general loading conditions.

To develop a successful design for constructing a ceramic dome, the primary concern was to find a suitable pattern for prestressing. In order to study different patterns, several small spheres were obtained upon which different designs could be sketched. Since there are an almost infinite number of configurations which could be studied, a decision was made to limit the investigation to designs which were based on the twenty equilateral spherical triangular segments of the spherical icosahedron.

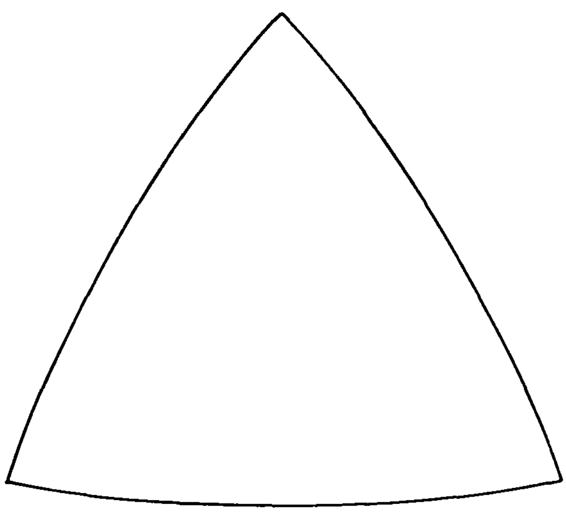
From this point, patterns were further developed by subdividing the equilateral spherical triangles in order to enable the approximation of a spherical shape by combining flat segments of still smaller size. The number of subdivisions was constrained by the following three factors:

1. Final segment size must be small enough for the assemblage to approximate a spherical shape.
2. The size of the largest and smallest triangles must not vary greatly.
3. The prestressing paths should approximate great circle arcs, and should avoid discontinuities in tangents to the paths as much as possible.

Preliminary calculations showed that a relatively good approximation of a sphere could be achieved by dividing each of the equilateral spherical triangles into thirty-six smaller triangles. This would mean that 180 elements would be needed to construct an icosacap, 360 for a hemisphere, and 720 for a complete sphere.

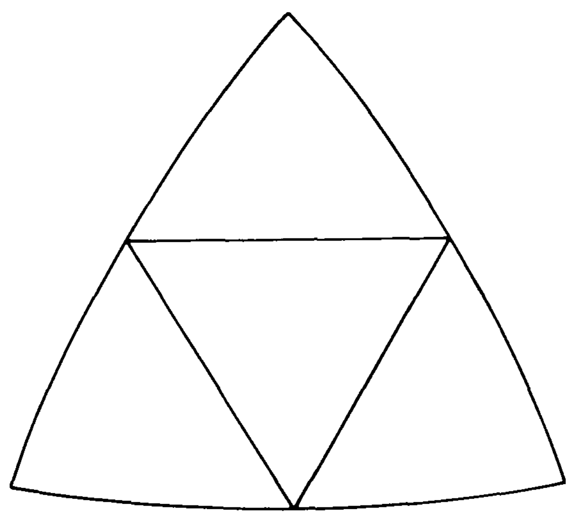
An accurately dimensioned twenty-inch plaster of paris hemisphere was then obtained upon which more detailed patterns could be studied. The first problem was the construction of the icosacap. Since it was impossible to project the lines of the icosahedron on the curved surface, it was necessary to calculate the points where the vertices of the icosahedron would be tangent to the surface of the sphere (see Appendix A). These points were laid out on the surface of the plaster sphere using a compass and a flexible batten.

To further subdivide the sphere, the midpoints of all sides of the spherical triangles (Figure 4) were connected to midpoints of adjacent sides using great circle arcs (Figure 5). Next, the center of each of the large triangles was found by constructing the bisector of each of the angles of the triangle and noting their point of intersection (Figure 6). From the center, six equal isosceles triangles were generated in such a manner that their edges were as nearly parallel to the edges of the large equilateral triangle as possible (Figure 7). By connecting the points where the isosceles triangles intersected the large equilateral triangle, three additional isosceles triangles were produced. Each of these nine isosceles triangles was further divided into four smaller isosceles triangles (Figure 8). Thus a complete sphere could be generated from 720 triangles of six different sizes.



SPHERICAL EQUILATERAL TRIANGLE

FIGURE 4



STEPS USED TO DIVIDE SPHERICAL EQUILATERAL TRIANGLES INTO SMALLER TRIANGULAR ELEMENTS

FIGURE 5

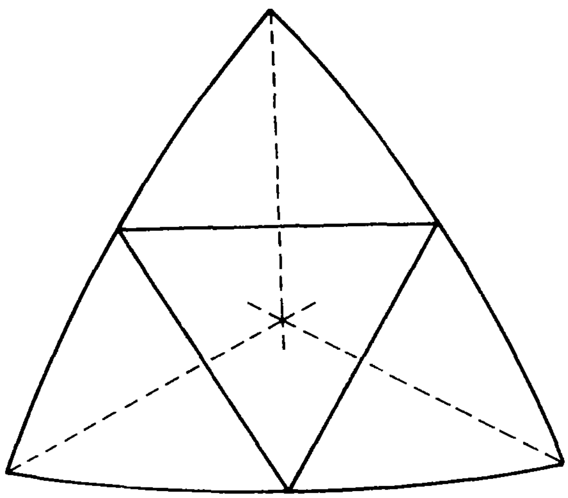


FIGURE 6

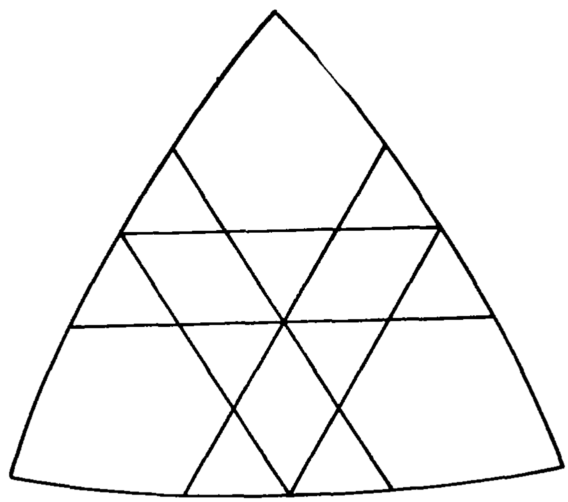


FIGURE 7

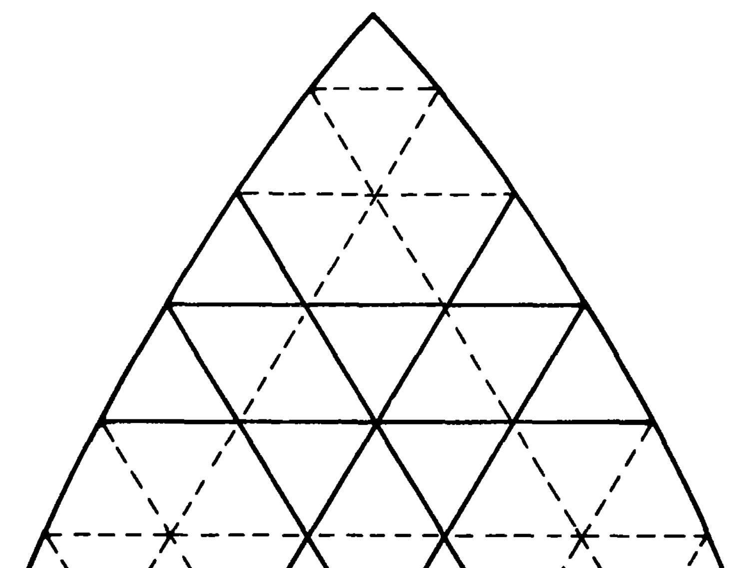
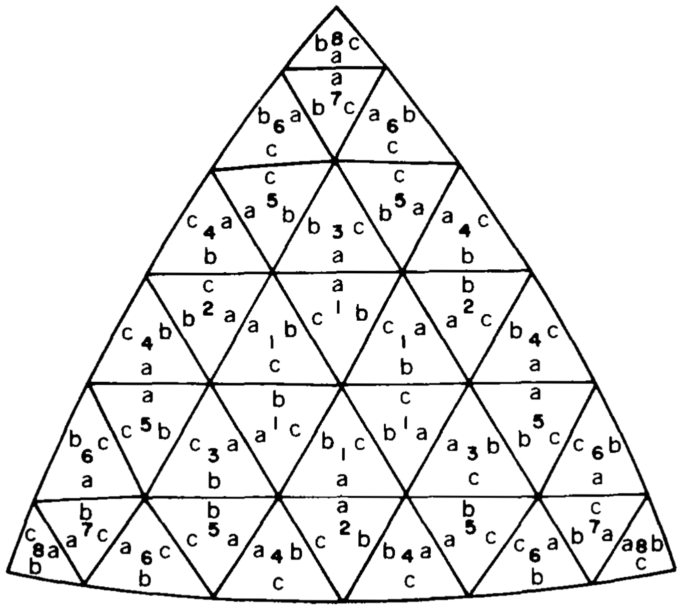


FIGURE 8

STEPS USED TO DIVIDE SPHERICAL EQUILATERAL TRIANGLES INTO SMALLER TRIANGULAR ELEMENTS

From this basic pattern the sizes of some of the triangles were altered in accordance with the three restrictions mentioned above. Although these changes increased the number of different triangles from six to eight, it is felt that the improvement in the prestressing pattern compensates for any additional complexity introduced. The final pattern (Figure 9) is by no means unique, and modifications will be incorporated as the need arises.



BASIC UNIT OF SPHERICAL DOME SHOWING ORIENTATION OF INDIVIDUAL ELEMENTS WITH RESPECT TO EACH OTHER

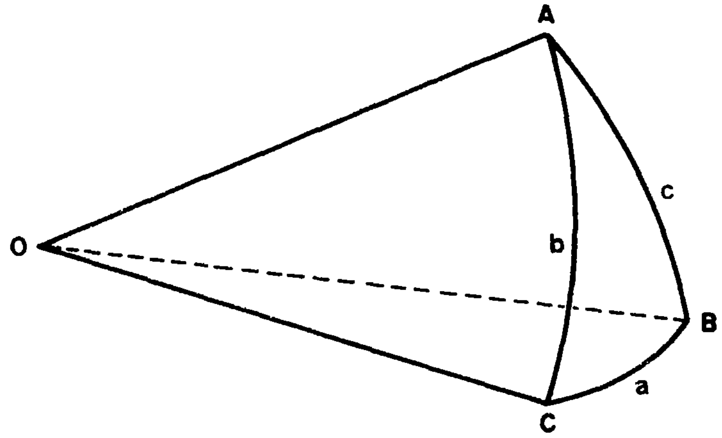
FIGURE 9

III. CALCULATION OF THE SIZE OF TRIANGULAR SEGMENTS

Once the prestressing pattern had been decided upon it was next necessary to calculate the size of the segments from which the sphere was to be constructed.

The first step was to construct, as accurately as possible, the prestress pattern as described in the previous section. This construction was done on a twenty-inch diameter plater of paris dome with all lines being part of great circle arcs.

The edges of all the smallest triangles were then measured to the nearest one-hundredth of an inch and the average length was obtained. Using this length as a basis it was possible to apply the sine and cosine laws of spherical trigonometry and obtain the spherical angles subtended by the arcs on the surface of the sphere. Referring to Figure 10, the sine law is given by



GEOMETRY OF A SPHERICAL SECTION

FIGURE 10

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} \tag{1}$$

and the cosine law by

$$\cos(a) = \cos(b) \cdot \cos(c) + \sin(b) \cdot \sin(c) \cdot \cos(A) \tag{2}$$

where

$a = \measuredangle\ COB$	$A = \measuredangle\ CAB$
$b = \measuredangle\ COA$	$B = \measuredangle\ ABC$
$c = \measuredangle\ AOB$	$C = \measuredangle\ BCA$

To find the actual edge lengths of each flat triangular segment it was necessary to apply the following equation (Figure 11):

$$\overline{AB} = \left[2\ r\ \sin\left(\frac{1}{2}\ c\right)\right] \tag{3}$$

The value for c was found from Equations (1) and (2).

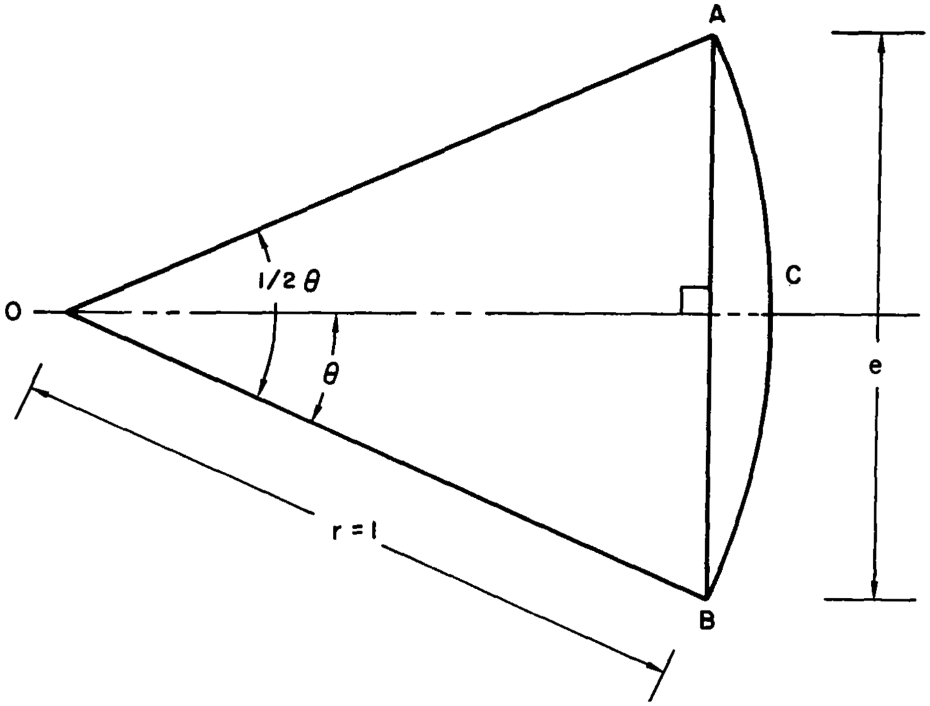
The included angles of the triangular segments were found from the same equation with the following transformation (Figure 12):

$$\overline{c} = \overline{AB} = 2b\ \sin\left(\frac{1}{2}\ \measuredangle\ ACB\right)$$

or

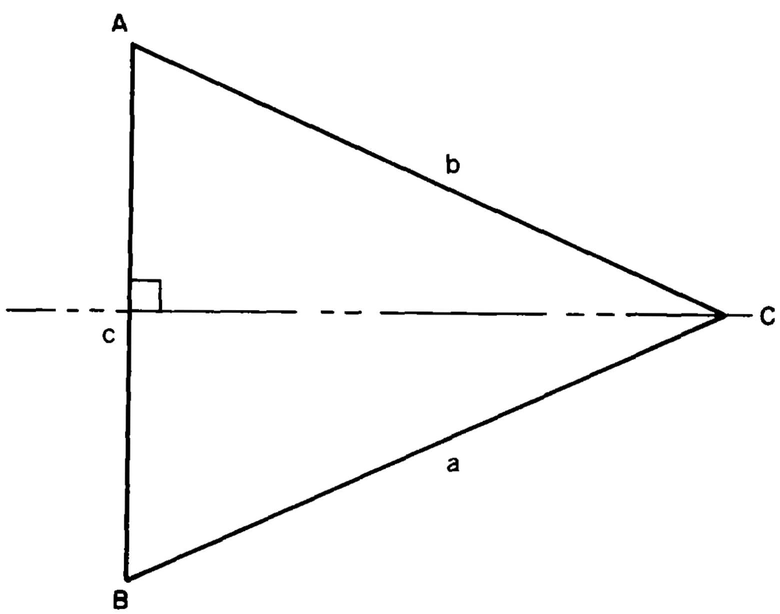
$$\measuredangle\ ACB = 2\ \sin^{-1}\left(\frac{1}{2}\ \frac{c}{b}\right)$$

The results of these calculations are presented in tabular form in Table B-1.



GEOMETRY OF AN ARC AND SUBTENDED ANGLE

FIGURE 11



GEOMETRY OF A TRIANGLE

FIGURE 12

IV. DESIGN OF SEGMENTS AND SPACERS

Several designs which would allow the prestressing cables to pass through the elements were considered before it was finally decided that it would be best, in an initial model, if the cables were to run between adjacent edges of the segments.

In order to transfer the force from the cable to the segment, cylindrical grooves would be cut along the edges of the triangles. A tube would be placed in the grooves between adjacent triangles and the prestressing wire passed through the tube. (Use of tubes was suggested by L. P. Felton.)

This tube would add shear strength to the interface between the triangles as well as provide a means of distributing the cable forces to the segments.

The plastic models were constructed of 3/8-inch plexiglass with 1/4-inch o.d. acrylic tubes used between the edges. To insure that no concentrated stress points would develop, the edges of the tubes were slightly beveled.

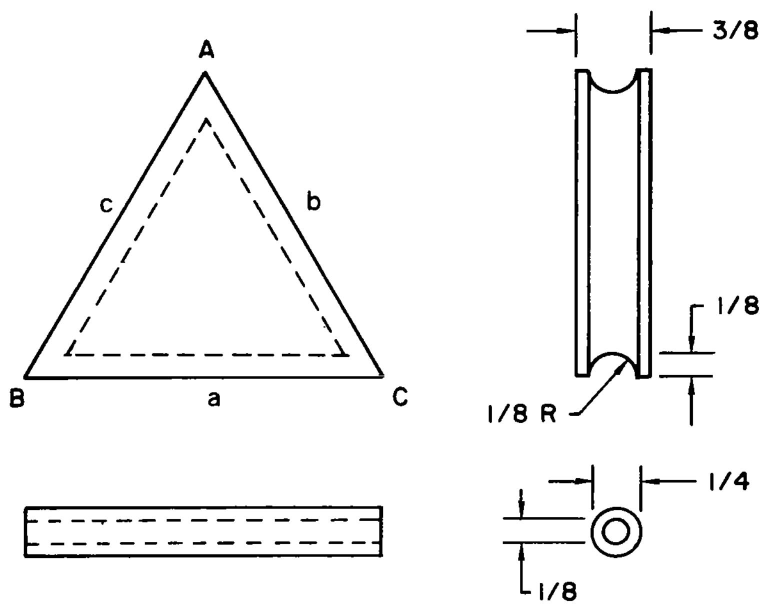
Figure 13 shows the general design of the segments and tubes, while Figure 14 shows an exploded view of typical assembled group of segments. Figure 15 and Table B-2 give the sizes of tubes required and their locations along the segment edges.

V. DESIGN OF END FIXTURES

The factors considered in designing the end fittings for the dome were as follows:

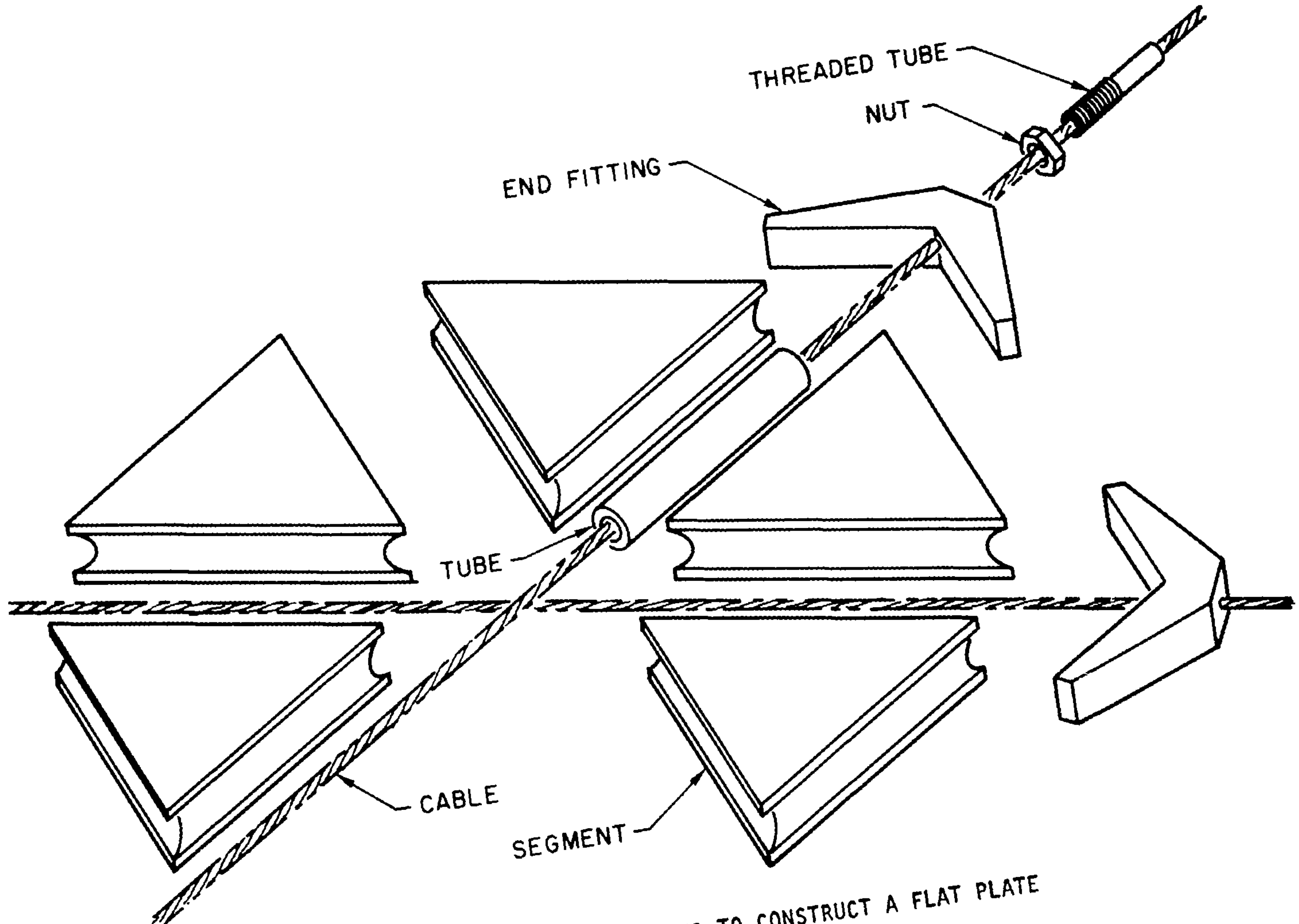
1. They must be lightweight and relatively compact.
2. They must be compatible with the ceramic (or plastic) structure.
3. They must provide a convenient method for applying the prestressing force.

The parts shown in Figures 16 through 19 give the dimensions and configurations decided upon in the final design. All parts were made of 2024-T4 aluminum alloy. In order to insure that the end fittings would be held securely in place, pieces of 1/4-inch half-round aluminum were bonded to the faces of the end fittings which came in contact with the outer edge of the dome. These served the same purpose as the tubes which were placed between the segments.



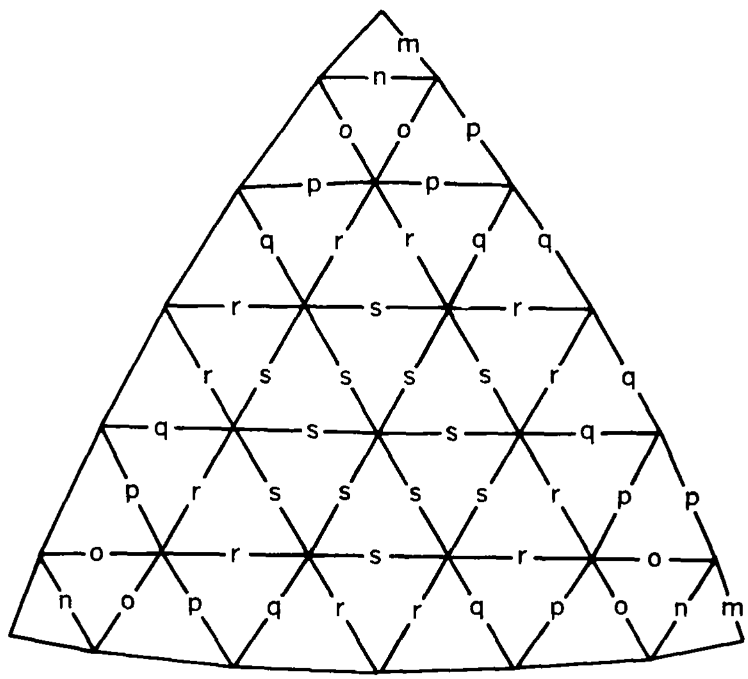
TYPICAL ELEMENT AND INTERLOCKING TUBE

FIGURE 13



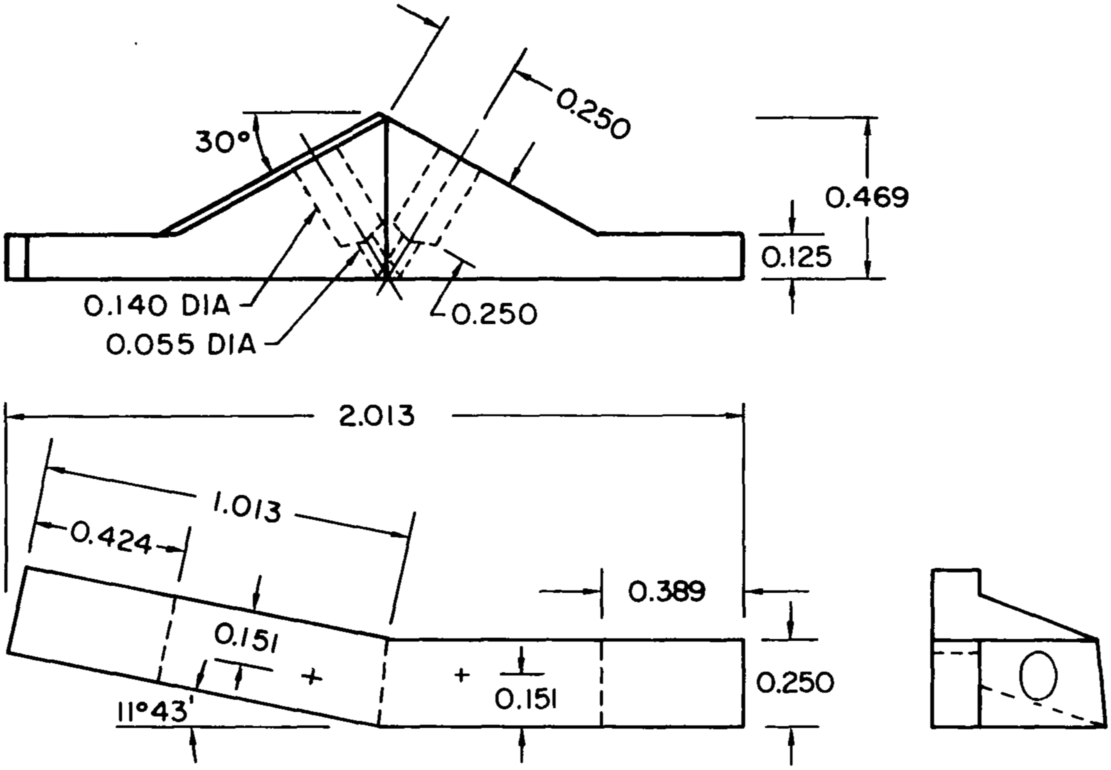
EXPLODED VIEW OF ELEMENTS USED TO CONSTRUCT A FLAT PLATE

FIGURE 14



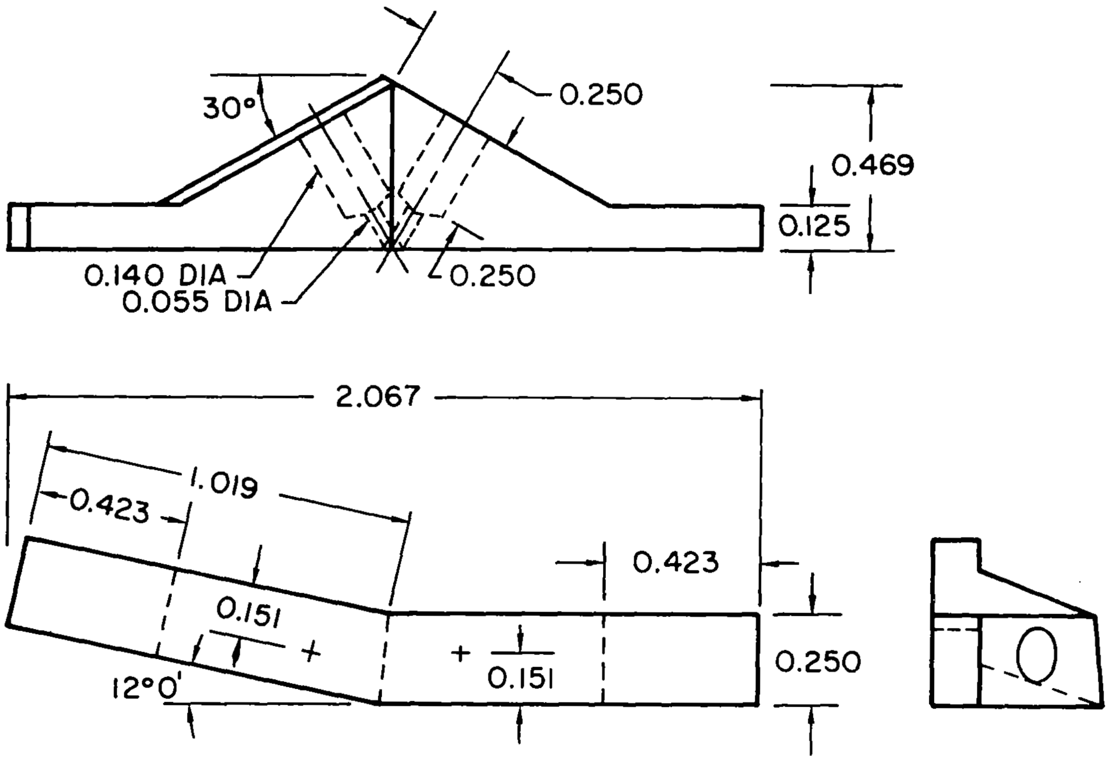
LOCATION OF INTERLOCKING TUBE WITH
RESPECT TO ELEMENT EDGES

FIGURE 15



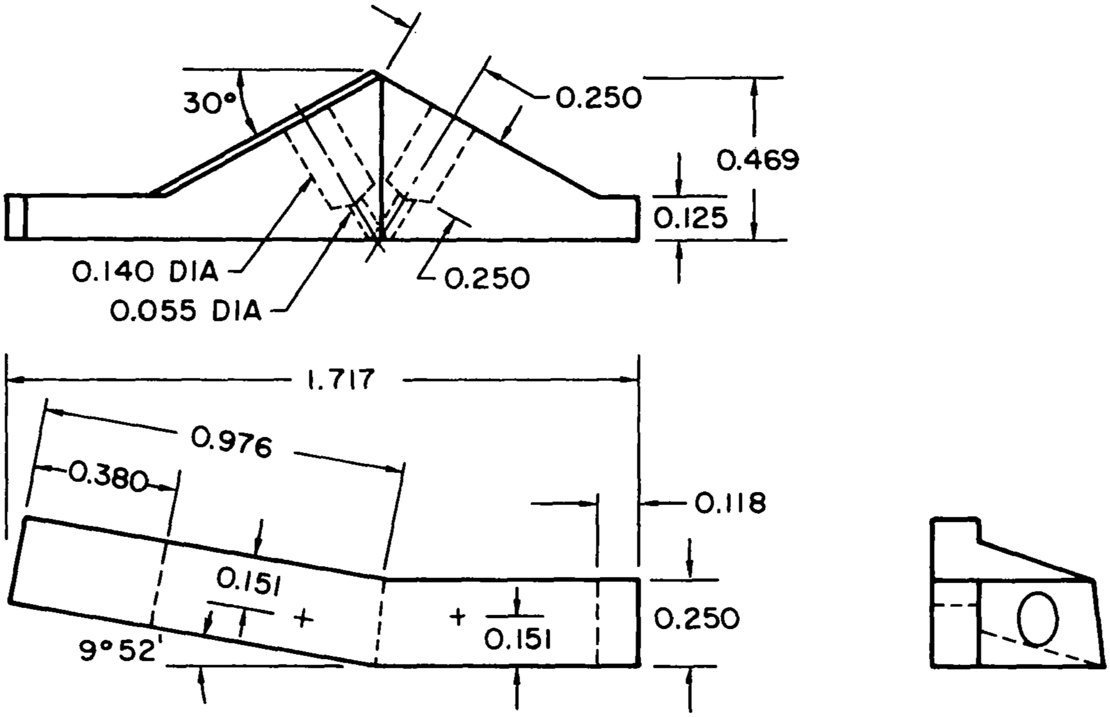
CONSTRUCTION DRAWING FOR END FITTING

FIGURE 16



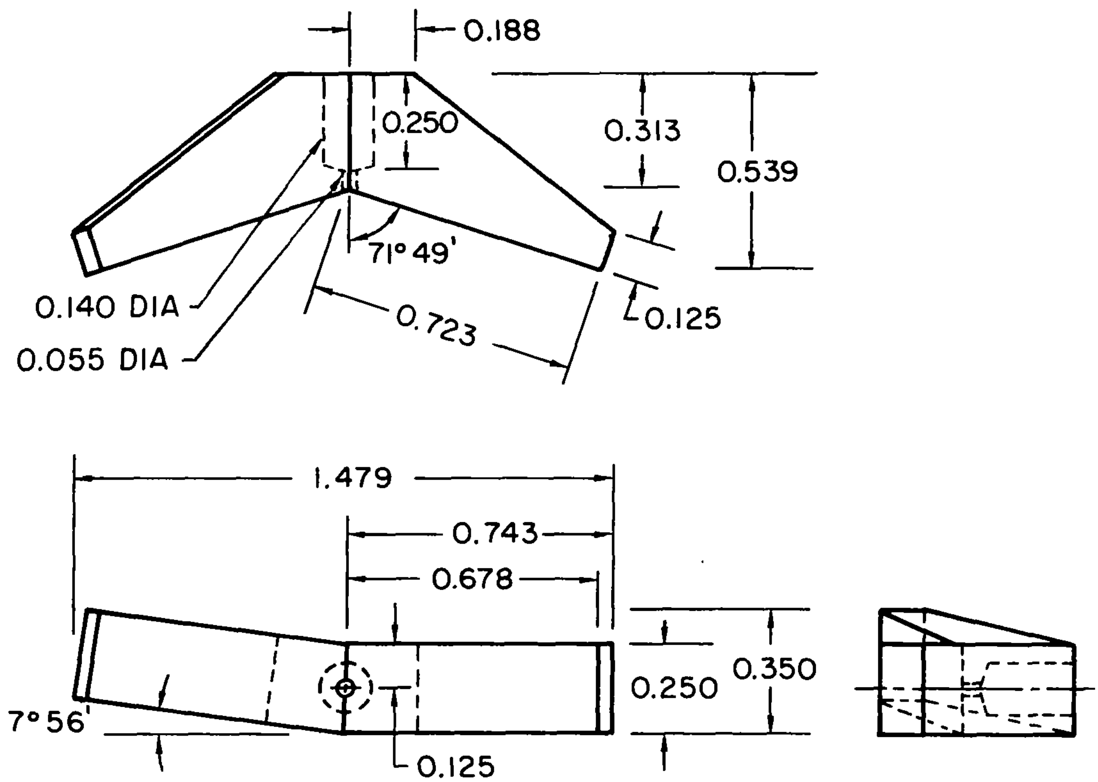
CONSTRUCTION DRAWING FOR END FITTING

FIGURE 17



CONSTRUCTION DRAWING FOR END FITTING

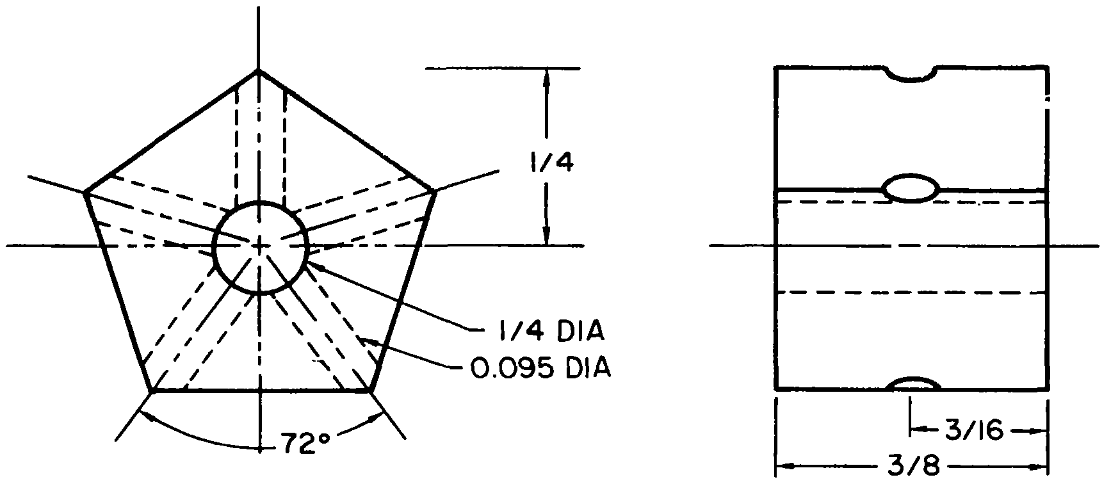
FIGURE 18



CONSTRUCTION DRAWING FOR END FITTING

FIGURE 19

It was believed undesirable to have five wires cross at the ap ex of the dome. A special fitting (Figure 20) was therefore constructed at which these wires could terminate.



APEX TERMINAL PLATE

FIGURE 20

VI. COMMENTS ON METHODS OF PRESTRESSING

Two methods of applying a prestressing force have been tried to date. The first, applicable primarily to flat plates, involved the use of carbon steel drill rod (0.049 in. dia.) which was cut to appropriate lengths. The ends were then threaded using a 00-90 die. The prestressing was achieved by tightening small brass nuts against the end fittings until the desired compressive force was produced in the structure. The major advantage in using the drill rod was the small size and weight of the hardware needed to transfer the prestressing force from the rod to the end fitting.

When construction was started on the dome, it was found from the following bending stress equation.[†]

[†]**Shanley**, F.R., *Strength of Materials*, pp. 277.

$$\sigma = \frac{y\,E}{R}$$

where: σ = stress

y = radius of rod

R = radius of curvature

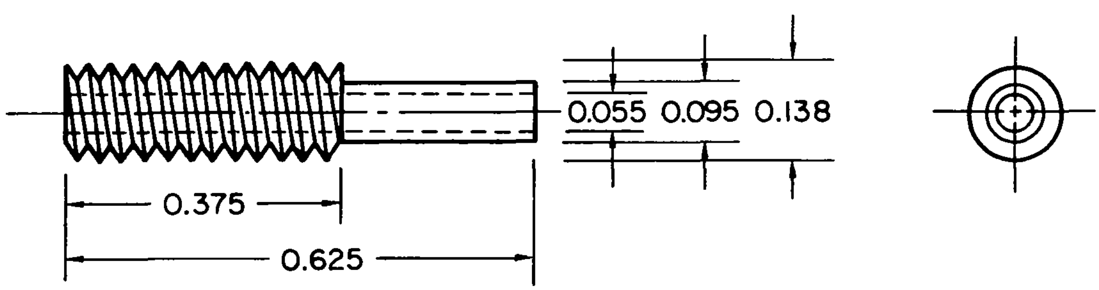
E = Young's modulus

that an outer fiber stress of ±51,000 psi would be induced as a result to making the rod conform to the geometry of the structure.

This bending stress would significantly reduce the prestressing load which could be applied. It was therefore decided that cable would be used because of its greater flexibility.

Several methods of attaching threaded fittings to the cable were tried and the results are summarized in Table C-1. It was found that the most secure attachment was achieved by first machining some threads off a 6 - 32 screw and drilling a hole through its center (see Figure 21). The cable was slipped through the hole and solder was allowed to flow between the walls of the tube and the cable. The fitting was then crimped to the wire by placing the threadless portion of the tube between vice jaws and applying a load sufficient to collapse the tube walls around the cable. The solder seemed to serve two purposes, 1) it increased the adhesive strength of the bond and 2) it kept the cable strands from being cut when the tube was crimped.

The prestressing force was applied in the same manner as described above.



PRESTRESS ADJUSTMENT FITTING

FIGURE 21

VII. ASSEMBLY

To hold the segments in place while the prestressing wires were being tightened, a 20-inch diameter concave hemisphere mold was obtained.

The edges of the segments were then beveled (to assure that the edges would not touch each other). The apex fitting and its five attached cables were put into place. The segments and cylindrical tubes were then placed in the mold in their respective positions (see Figures 9 and 15). Finally, the end fixtures were laid into place after all the segments had been aligned.

The threaded fittings were next attached to one end of each of the prestressing wires, and the wires were threaded through the tubes. The remaining threaded fittings were slipped onto the free end of the cables and soldered in place. The entire structure was subjected to a compressive load by tightening the cables as described in the previous section.

VIII. SUMMARY AND PROJECTIONS FOR FUTURE WORK

The construction of the plastic icosacap demonstrated that the calculations of the geometry were correct to the degree required for the accurate fabrication of a segmented ceramic shell. It also provided the opportunity to try various methods of prestressing and design of end fittings. Threaded wire cables and aluminum support fittings proved to be suitable for this project.

Some of the additional subjects for future research in this area are:

1. Optimum design of groove and segment thickness (includes possible use of curved grooves).
2. Design for grooves <u>without</u> tubes (necessary for flat or developable configurations).
3. Optimum design of tubes (inside and outside diameters) and also the shape of the tube which allows the cables to transmit forces as uniformly as possible.
4. Development of a ceramic body with suitable strength properties at both low and high temperatures.
5. Development of a prestressing cable which will retain its strength properties at elevated temperatures, and show thermal expansion characteristics similar to the ceramic used to fabricate the segments. (Work is now underway on ceramic cables.)

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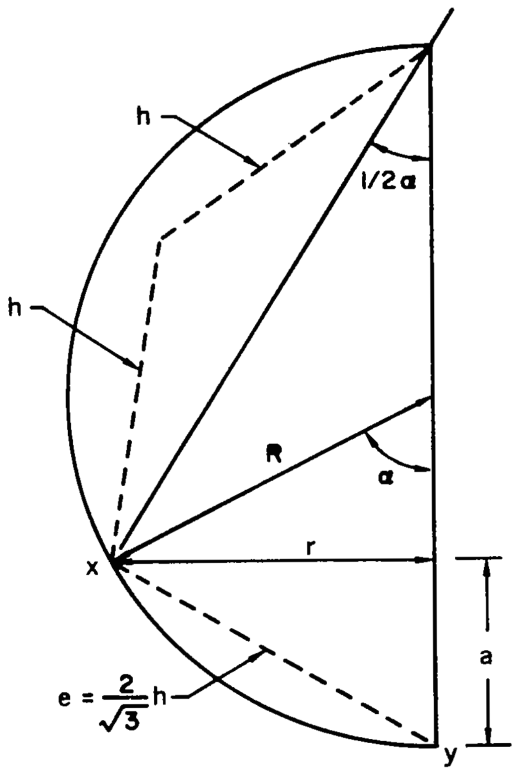
APPENDIX A

TABLE A-1

SUMMARY OF CALCULATIONS USED TO CONSTRUCT THE ICOSACAP ON THE PLASTER HEMISPHERE



R = Radius of sphere

h = Height of triangle of inscribed icosahedron

e = Edge of triangle
= 1.15470 h
= 1.05145 R

r = Radius of circle circumscribed about a pentagon with edges of length 1.15470 h
= 0.98225 h

Length of arc xy = 1.10723 R

$\alpha = 63° \; 26' \; 20''$

APPENDIX B

TABLE B-1

SUMMARY OF CALCULATIONS OF ELEMENT SIZES

Element No.	A	B	C	a	b	c	No. Required for Hemisphere
1	59° 30'	60° 15'	60° 15'	2.150	2.166	2.166	60
2	62° 54'	58° 33'	58° 33'	2.150	2.060	2.060	30
3	61° 42'	59° 09'	59° 09'	2.150	2.096	2.096	30
4	57° 56'	60° 17'	61° 47'	2.010	2.060	2.090	60
5	58° 48'	63° 08'	58° 04'	2.010	2.096	1.994	60
6	55° 22'	62° 19'	62° 19'	1.853	1.994	1.994	60
7	54° 30'	62° 45'	62° 45'	1.696	1.853	1.853	30
8	71° 48'	54° 06'	54° 06'	1.696	1.446	1.446	30

REFER TO FIGURE 13.

NOTE: Dimensions a,b, and c are ratios and should be multiplied by 0.1 times the mean radius of the sphere to obtain the correct element sizes.

TABLE B-2

SUMMARY OF TUBE SIZES

Tube Designation	Length	No. Required
m	1.040	10
n	1.225	15
o	1.400	30
p	1.550	40
q	1.675	40
r	1.626	60
s	1.725	60

REFER TO FIGURE 15 FOR LOCATION OF TUBES.

APPENDIX C

TABLE C-1

TESTS ON FIXTURES USED FOR CABLE PRESTRESSING

Methods of crimping:

a) Concentrated load is applied at three points by placing 1/16" dowels between vice jaws and the tube to be crimped.

b) A distributed load is applied along the entire length of the unthreaded portion of the tube by squeezing it directly with the vice jaws.

Test Results:

Test No.	Wire Dia.	Wire Config.	Failure Load	Failure Mode	Method of Crimp	Remarks
1	1/32	7x3	35 lb	wire slipped through end fitting	a	
2	1/32	7x3	42 lb	part of wire broke at crimp, remaining part slipped through	b	excess crimping sheared part of wire
3	1/32	7x3	93 lb	wire broke at crimp	b	end of wire was soldered after crimping
4	1/32	7x3	96 lb	wire broke at crimp	b	brass fitting used instead of steel
5	3/64	7x7	276 lb	wire broke away from crimp	b	solder was allowed to flow between wire and tube before crimping. End of wire was also soldered after crimping

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